Asynchronous Execution and Communication Latency in Distributed Constraint Optimization

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Outline:

• Motivation: real-time coordination of sensors in a high-latency network
• Modeling coordination as graph colouring
• Soft graph colouring for real-time responsiveness
• A class of distributed anytime algorithms (synchronous)
• Convergence
• Tightness of constraints: conservative variant
• Scalability and robustness
• Asynchronous execution
• Very high communication latencies

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Motivation: Large Networks of Short-Range Sensors

- **Short-range, directional radars**
  - each can scan 1 of its 3 sectors at a time
  - each scan acquires range & radial velocity
  - battery-operated – conservation important

- **Collaboration needed for tracking**
  - 3 approximately-simultaneous scans needed for trilateralization

- **Low-power radio communication**
  - low bandwidth, high latency
  - reveals positions of radars – minimize

- **Coordination mechanism** organizes collaboration
  - optimizes simultaneous scanning, minimizes costs

- **Must be:**
  - scalable (e.g., to $10^5$ sensors)
  - real-time adaptive (e.g., new targets are detected, existing targets disappear)
  - robust (e.g., hardware may fail)
Inter-Sensor Collaboration

- **Main requirement:** scan each target simultaneously with 3 radars
  - define virtual resources: *trackers*
  - each tracker is comprised of 3 sectors on nearby radars
    - $T_i \equiv \{R_{i1}:S_{i1}, R_{i2}:S_{i2}, R_{i3}:S_{i3}\}$
    - each tracker can track a single target over some contiguous region
- **Main constraint:** each radar can scan only 1 sector at a time
  - if two trackers use different sectors on the same radar, they are mutually exclusive
    - $\text{mutually}\_\text{exclusive}(T_1, T_2) \iff \exists \ j,k \in \{1, 2, 3\}: R_{1j} = R_{2k} \land S_{1j} \neq S_{2k}$
- **Compute a cyclic schedule of tracker usage**
  - worst-case assumption: all trackers need to be used
  - mutually exclusive trackers cannot be used in the same time slot
  - number of time slots determined by target speed, scan time & revisit period

<table>
<thead>
<tr>
<th>timeslot #</th>
<th>scan start time (seconds)</th>
<th>scan end time (seconds)</th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
<th>T4</th>
<th>T5</th>
<th>T6</th>
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<td></td>
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<td>X</td>
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<td>X</td>
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</tr>
<tr>
<td>3</td>
<td>4.0</td>
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<td>X</td>
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<tr>
<td>4</td>
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<td>8.0</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
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</table>
Modeling Coordination as Graph Colouring

- Each tracker can be mapped to a node in an undirected graph.
- Each mutual exclusion constraint then maps to an edge:
  - Nodes that are adjacent in the graph are mutually exclusive/cannot be used simultaneously.
  - Two nodes are said to be neighbors iff they are adjacent.
- A proper k-colouring of the graph’s nodes maps to a feasible schedule:
  - Time slot $\Leftrightarrow$ integer in $\mathbb{Z}_k \Leftrightarrow$ colour.

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<tr>
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<th>scan start time (seconds)</th>
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<th>T1</th>
<th>T2</th>
<th>T3</th>
<th>T4</th>
<th>T5</th>
<th>T6</th>
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<td></td>
<td></td>
</tr>
<tr>
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<td>2.0</td>
<td>4.0</td>
<td></td>
<td>Yellow</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4.0</td>
<td>6.0</td>
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<td></td>
<td>Cyan</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>6.0</td>
<td>8.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Green</td>
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</tr>
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</table>
Soft Graph Colouring

• An edge connecting nodes of the same colour represents a *conflict*
  – some radar has been scheduled to scan two sectors simultaneously

• For real-time adaptation, the number of conflicts must be quickly reduced
  – fast reduction to acceptable levels is more important than total elimination

• Define the *degree of conflict* as the fraction of edges that are conflicts
  – let $E$ be the set of edges and $C_v$ the colour of node $v$

  $\gamma \equiv \frac{|\{u, v\} \in E \mid C_u = C_v|}{|E|}$

  $0 = \text{proper colouring}$
  $1/k = \text{random } k\text{-colouring}$
  $1 = \text{single-colour colouring}$

• Normalize: $\Gamma \equiv k\gamma$
  – random $k$-colouring has an expected $\Gamma$ of 1

• Assessment of coordination mechanism is based on how quickly it reduces $\Gamma$ after random initialization
A Class of Distributed Anytime Algorithms (synchronous)

• Main idea: each node repeatedly chooses its own colour to minimize its conflicts with neighbouring nodes

• Fixed Probability algorithm FP(p) …
  – Initialization:
    • each node chooses a random colour and informs its neighbours
  – Synchronized infinite loop:
    • probabilistic activation
      – a node activates if a randomly generated number falls below some fixed activation level p
    • if a node activates, it non-deterministically chooses its next colour
      – it computes a histogram of colour usage among its neighbours, based on what they last told it
      – it then chooses any colour that is least used in the histogram
      – if the chosen colour differs from its current colour, it tells its neighbours

• Convergence?
  – under the right conditions, the total number of conflicts reduces over time and may converge to 0 …
Effect of Activation Level on Convergence of FP

- Measure (normalized) degree of conflict after each synchronous step
  - experiment performed in simulator
- When activation level is too high, thrashing occurs
  - too many neighbours are simultaneously updating colours
  - because of out-of-date information, they make mutually harmful decisions
- When activation level is too low, adaptivity is hindered
  - extreme case is sequential execution
- Need compromise between speed and coherence
  - an activation level of 0.3 seems to be reasonable for sparse graphs
  - this level was used for experiments reported in following slides

- experimental results shown for 2D grids
  - number of colours = chromatic number = 4
  - 500-5000 nodes
- experiments also performed with random graphs having higher, known chromatic numbers
Animation: Activation Threshold

2DX · FP10% · 4 colors
Step 0000: initialization
Conflicts: 25.2%

2DX · FP30% · 4 colors
Step 0000: initialization
Conflicts: 25.0%

2DX · FP90% · 4 colors
Step 0000: initialization
Conflicts: 25.0%

2DX · FP50% · 4 colors
Step 0000: initialization
Conflicts: 24.7%
Effect of Tightness of Constraints

- Performance of FP is good on over-constrained problems
  - where #colours < chromatic number
  - for 2D & 3D grids, observed convergence value of degree of conflict is close to theoretical minimum
- Performance of FP is poor on loosely constrained problems
  - where #colours >> chromatic number
  - intuitively, these are easy problems
- When loosely constrained, each colour choice is essentially random
  - for each given node, most colours are not used by any neighbour
  - FP chooses randomly from among the unused colours
  - asymptotic value predicted as $\alpha/(2-\alpha)$ where $\alpha$ is the activation level

This is not a time axis

- experimental results shown for 2D grids
- chromatic number = 4
Animation: Tightness of Constraints

<table>
<thead>
<tr>
<th>Constraint Level</th>
<th>Conflicts</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 colors</td>
<td>49.7%</td>
</tr>
<tr>
<td>3 colors</td>
<td>33.0%</td>
</tr>
<tr>
<td>4 colors</td>
<td>24.7%</td>
</tr>
<tr>
<td>6 colors</td>
<td>19.3%</td>
</tr>
<tr>
<td>12 colors</td>
<td>08.6%</td>
</tr>
<tr>
<td>10 colors</td>
<td>09.8%</td>
</tr>
<tr>
<td>8 colors</td>
<td>12.5%</td>
</tr>
<tr>
<td>6 colors</td>
<td>16.8%</td>
</tr>
</tbody>
</table>
CFP: Conservative Variant

- Colour choice is non-deterministic
- But activation is restricted
  - in addition to passing the test for random number<activation level, a node may activate only if it has a conflict with any neighbour
- Conservative variant has good performance overall
  - communication costs are also better than FP’s for loosely constrained problems
    - under FP, node activity continues unabated forever
    - under CFP, node activity decreases with the degree of conflict

- experimental results shown for 2D grids
- chromatic number = 4
Animation: FP vs. CFP

- 2DX - FP30% - 5 colors
  - Conflicts: 19.7%

- 2DX - CFP30% - 5 colors
  - Conflicts: 20.3%

- 2DX - FP30% - 6 colors
  - Conflicts: 16.7%

- 2DX - CFP30% - 6 colors
  - Conflicts: 16.2%

- 2DX - FP30% - 8 colors
  - Conflicts: 12.2%

- 2DX - CFP30% - 8 colors
  - Conflicts: 12.6%

- 2DX - FP30% - 10 colors
  - Conflicts: 09.4%

- 2DX - CFP30% - 10 colors
  - Conflicts: 09.9%
Scalability

• The algorithm is scalable in cost
  – per node, per step costs depend on (mean) degree of the graph
  – they do not depend on the number of nodes
    • to the extent that the mean degree is independent of the number of nodes
• The algorithm is scalable in performance
  – for large graphs, the reduction in normalized degree of conflict over steps shows little variation for graphs of different sizes

• results shown are for CFP(0.3)
• 6 graphs of different sizes (500-5000 nodes)
  – each graph has chromatic number 4
  – each was coloured using 2, 3, 4 & 5 colours
Robust against Communication Noise

- Each colour-change message subjected to random process:
  - probability $r$, colour randomized
  - probability $d$, message lost
  - otherwise, message unchanged
- For small amounts of noise, incremental increases in degree of conflict are observed
  - no catastrophic failure

- results shown are for CFP(0.3) on 2D grids with 4 colours subject to various amounts of message randomization
- similar results were obtained for small amounts of message loss
Asynchronous Execution

• The synchronous FP algorithm requires synchronization, which may:
  – require overhead (e.g. communication cost)
  – slow down the process (wait for the slowest message and node)
  – slow down convergence — or not

• For asynchronous FP the essential idea is the same as for synchronous version, except that execution is asynchronous:
  – Non-synchronized infinite loop (but same rate for all nodes):
    • probabilistic activation
      – a node activates if a randomly generated number falls below some fixed activation level $p$
    • if a node activates, it non-deterministically chooses its next colour
      – it computes a histogram of colour usage among its neighbours, based on what it last heard from them
      – it then chooses any colour that is least used in the histogram
      – if the chosen colour differs from its current colour, it tells its neighbours

• Asynchrony may help in symmetry breaking, but communication latency may cause ill-advised changes
Effect of Communication Latency

- Performance of asynchronous FP is reasonable for moderate latencies
  - short-term performance degrades (as expected)
  - long-term result quite good
- Performance is even better than synchronous FP when latency < 0.5 time units
- Performance sharply becomes very poor for higher latencies
  - divergence
  - latency = 7 not better than random colouring

- experimental results averaged for 20 random graphs
- $p = 0.3$
- mean degree = 10
- chromatic number = 3
Communication Latency and Activation Probability

• Sharp performance drop for higher latencies: the threshold latency decreases as activation probability increases

• This is due to higher probability of "collision": a colour-change message still travelling along an edge when decision is taken

• degree of conflict averaged over 10,000 steps
• mean degree = 10
• chromatic number = 3
Effect of Collision Probability

- For activation probability $p$ and latency $L$, (an upper bound on) the probability of collision is about $1 - (1 - p)^L$
- Performance drop indeed depends on collision probability: fine up to about 0.8; bad at 0.9 and higher
- So given latency $L$, a safe activation probability is:
  
  $p \leq 1 - 0.2^{1/L}$

<table>
<thead>
<tr>
<th>$L$</th>
<th>$p \leq$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.80</td>
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<tr>
<td>2</td>
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<tr>
<td>4</td>
<td>0.42</td>
</tr>
<tr>
<td>8</td>
<td>0.18</td>
</tr>
</tbody>
</table>

- degree of conflict averaged over 10,000 steps
- mean degree = 10
- chromatic number = 3
**Very High Latencies**

- Surprise: for very high latencies, the normalized degree of conflict $\Gamma$ tends to a mean value of approximately 2
- $p = 0.3$
- $L = 15$

- For very high latencies, the control mechanism gets caught in an out-of-phase, oscillating trajectory, with period $> 2L$
- $p = 0.3$
- $L = 10$
Conclusion

- The FP algorithm is simple but effective for distributed, real-time, approximate colouring of sparse graphs
  - scalable, low-cost, robust
- Basic framework of stochastic activation & local optimization seems appropriate for other distributed constraint problems
  - graph colouring serves as a clean, archetypal problem
- The algorithm has also been tested with dense, random graphs
  - interesting, but different, results
  - proper k-colourings quickly obtained for very dense k-colourable graphs
    - local constraints guide colouring to a unique, proper colouring
- Asynchronous execution and communication latency are handled well
  - provided that the activation probability does not exceed a critical level
- Further work on algorithm
  - non-uniform activation levels, perhaps determined dynamically from local metrics